Review of Partial Fractions

The purpose of the method of partial fractions is to express a fraction having a complicated polynomial denominator as the sum of fractions whose denominators are simpler polynomials.

Step 0. Make sure that the degree of the numerator is strictly less than the degree of the denominator. If the degree of the numerator is equal or greater, use long division before using partial fractions.

Step 1. Choose the form of the partial fraction decomposition.

(a) Factor the denominator into linear and irreducible (non-factorable) quadratic factors.

(b) Each repeated factor in the denominator requires multiple fractions in the decomposition. For example, the expression $(x-1)^3$ in the denominator requires three fractions — one with denominator $(x-1)^3$, one with denominator $(x-1)^2$, and one with denominator $x-1$.

(c) Each denominator which is a linear polynomial or a power of a linear polynomial requires a constant numerator.

(d) Each denominator which is an irreducible quadratic polynomial or a power of an irreducible quadratic polynomial requires a linear numerator.

Example. $\frac{P(x)}{(x+3)(x-1)^3(x^2+1)^2} = \frac{A}{x+3} + \frac{B}{(x-1)^3} + \frac{C}{(x-1)^2} + \frac{D}{x-1} + \frac{Ex+F}{(x^2+1)^2} + \frac{Gx+H}{x^2+1}$

Step 2. Work out the constants.

Example. $\frac{6}{x^2-6x+8} = \frac{6}{(x-2)(x-4)} = \frac{A}{x-2} + \frac{B}{x-4}$

$6 = A(x-4) + B(x-2)$

Strategy 1. Equate coefficients of each power of x.

$0 = A + B \qquad A = -3$
$6 = -4A - 2B \qquad B = 3$

Strategy 2. Cleverly choose values for x which will simplify the equation.

Let $x = 2$: $\quad 6 = -2A \qquad A = -3$
Let $x = 4$: $\quad 6 = 2B \qquad B = 3$

Therefore $\frac{6}{(x-2)(x-4)} = \frac{-3}{x-2} + \frac{3}{x-4}$.